The Deputy Secretary of Energy
Washington, DC 20585

July 29, 2011

The Honorable Peter S. Winokur
Chairman
Defense Nuclear Facilities Safety Board
625 Indiana Avenue, NW, Suite 700
Washington, DC 20004

Dear Mr. Chairman:

Thank you for your April 8, 2011, letter regarding technical and software quality assurance issues with the computer program System for Analysis of Soil-Structure Interaction (SASSI). SASSI is widely used within the Department of Energy (DOE), as well as in the nuclear industry, to analyze the effect of seismic ground motions on structures, and its outputs can play a key role in the seismic design of facilities.

Enclosed you will find the response report entitled U.S. Department of Energy Report on Technical and Software Quality Assurance Issues Involving the System for Analysis of Soil-Structure Interaction. This response report conveys DOE’s current understanding of the problems identified. DOE will continue to work with the Defense Nuclear Facilities Safety Board (Board) and its staff as the schedule evolves to identify any additional appropriate actions. This response report provides background on the SASSI code, the problems with the code’s subtraction method, and the steps DOE has taken, and is taking, to address the technical and software quality assurance issues discussed in your April 8, 2011, letter. This response report was developed with input from the National Nuclear Security Administration (NNSA), the Office of Environmental Management (EM), and the Office of Health, Safety and Security (HSS), all of which are committed to implementing actions to address the SASSI concerns detailed therein.

DOE has been working to understand the causes and impacts of the problems with the SASSI subtraction method ever since learning of this issue in the summer of 2010. In this regard, DOE is distributing to field elements a technical report entitled U.S. Department of Energy Soil-Structure Interaction Report, July 2011. This report, which is also enclosed, provides background on the subtraction method problems, recommendations for reviewing past SASSI subtraction method analyses, and advice on avoiding subtraction method errors in future analyses.

DOE managers have benefitted from meeting and working with your technical staff on this issue, and we will continue to seek the Board’s expertise and advice. Because SASSI is also widely used outside of DOE, we are sharing the enclosed technical report with...
organizations including the Nuclear Regulatory Commission, Nuclear Energy Institute, and the Institute of Nuclear Power Operations. We will continue to distribute future findings related to any SASSI issues to interested parties within and outside of DOE.

Mr. Richard Lagdon, Chief of Nuclear Safety, and the responsible personnel within NNSA, EM, and HSS will brief you and your staff on this matter.

If you have any questions, please contact me or Mr. Lagdon at (202) 586-0799.

Sincerely yours,

Daniel B. Poneman

Enclosures
Background

System for Analysis of Soil-Structure Interaction (SASSI) is a computer code for performing finite element analyses of soil-structure interaction during seismic ground motions. The code is widely used in the nuclear industry. SASSI was first developed in 1981 at the University of California at Berkeley (UC Berkeley), and several modified, proprietary versions are now available. In the early years, SASSI was commonly executed with a flexible volume method, also known as the direct method, in which every finite element node within and on the perimeter boundary of the excavated soil volume is treated as an interaction node that couples the free-field soil system and the excavated soil volume. In 1998, a more computationally efficient method known as the subtraction method was developed for SASSI execution (Chin, 1998). In the subtraction method, only the nodes on the outer perimeter boundary are treated as interaction nodes (see Figure 1). The most recent user's manual for the SASSI2000 version of the code states that the subtraction method is the preferred method of analysis.

Figure 1: Comparison of finite element mesh construction for SASSI direct and subtraction methods.

In 2010, analyses revealed that the subtraction method, under some conditions, provides results that deviate significantly from those of the direct method (Mertz et al., 2010). The inconsistent results occur at ground motion frequencies above that of the one-dimensional natural frequency of the excavated soil volume. The subtraction method has been found to both overestimate and underestimate the seismic response, depending on the frequency of interest.
Upon learning of the Mertz et al. work, the Department of Energy (DOE) Chief of Nuclear Safety (CNS) initiated an investigation into the issue for developing recommendations on addressing the issue at existing facilities that have used SASSI in the past, as well as facilities still under design using SASSI. This investigation included interfacing with several experts in soil structure interactions. These individuals have considerable experience in designing finite element meshes for SASSI, executing the code, and processing results.

Through late 2010, DOE created and analyzed sample problems, and communicated with SASSI experts, in attempts to isolate the problem with the subtraction method. DOE evaluated a modified subtraction method and its efficacy at avoiding the shortcomings of the subtraction method. A set of test problems was developed to identify the subtraction method limitations. DOE compiled these test problems into a technical report, *U.S. Department of Energy Soil-Structure Interaction Report*, July 2011, which provides background on the subtraction method problems, evidence of the robustness of the modified subtraction method, recommendations for reviewing past SASSI subtraction method analyses, and advice on avoiding subtraction method discrepancies in future analyses. The technical report is being transmitted to the affected sites in July 2011; it is being provided to the Defense Nuclear Facilities Safety Board (Board) staff along with this report.

On January 19, 2011, DOE met with the Board staff to discuss the SASSI problems, activities and progress to date, and planned activities to address the problems. On April 8, 2011, the Board submitted to DOE a letter and technical report to express its concerns with SASSI technical and software quality assurance issues and to request from DOE a report and briefing on how the Department intends to address these concerns. This report responds to the Board letter, and was developed with input from the National Nuclear Security Administration (NNSA), the Office of Environmental Management (EM), and the Office of Health, Safety and Security (HSS).

**Issues Identified by the Board**

The April 8, 2011, letter listed five specific issues the Board would like to see addressed in this report. These issues, and the commitments by DOE to implement actions to address them, are detailed below.

1) **Address the need for a root cause analysis of the SASSI issues**

At this time, the only clearly defined technical issue with the SASSI code is the problem with the subtraction method, for which we have conducted an analysis to determine the extent of condition. DOE is confident that the conditions under which subtraction method results are unreliable are well defined. In short, under some site conditions and excavation geometries, the subtraction method yields ground motion transfer functions with unacceptable deviations compared with the more reliable direct method results. This technical problem appears to be common to all variations derived from SASSI2000. DOE has found that in general, the direct and subtraction methods diverge when three conditions coincide:

1) the structures are embedded;
2) the structures have wide, shallow foundations; and
3) the structural response frequencies are close to, or higher than, the first mode frequency of the excavated soil volume.

These findings agree with the results of Mertz et al. (2010). DOE’s technical report, U.S. Department of Energy Soil-Structure Interaction Report, includes 11 example problems that help define the bounds of reliability of the subtraction method and demonstrate the efficacy of the modified subtraction method, first described by Mertz (2010). The modified subtraction method provides agreement with the direct method results over a larger frequency range by adding interaction nodes to the finite element mesh (FEM) created for the analyses.

The transfer function discrepancies between the direct and subtraction methods commonly occur close to the first mode frequency of the excavated soil volume. As a result, the technical report concludes that this frequency should be considered the limit of reliability of the subtraction method. Subtraction method discrepancies also occur near the FEM maximum transmission frequency, also known as the cutoff frequency, but DOE has not discerned a relationship between the magnitude of the discrepancies and the FEM characteristics. This latter problem can be avoided through proper element sizing in the FEM. Building the FEM with a top layer of interaction nodes corresponding to the ground surface of the excavated volume—defined as the modified subtraction method—largely eliminates transfer function discrepancies as compared to the direct method. Discrepancies sometimes still exist with the modified subtraction method, but only at higher frequencies. As the number of interaction nodes in the FEM increases, the frequency of the excavated volume also increases. To ensure reliable results, a FEM must be constructed to ensure that the first mode frequency of the excavated volume is higher than the highest frequency of interest in the response analysis. The DOE technical report describes this in further detail.

The modified subtraction method will be described in supplemental guidance developed by DOE. This supplemental guidance is addressed in Action 7. DOE believes that additional sample problems will be beneficial for DOE SASSI users to validate and verify the code for use at their sites. These tasks are among those described in DOE’s plans for future work, discussed under Issue 4 and Action 6.

DOE’s subtraction method analyses also found that discrepancies are likely to have less impact at Western U.S. sites. Western U.S. input motion time histories tend to have low energy content at high frequencies, so any divergence in a transfer function derived with the subtraction method will have little impact on the resulting response spectrum.

A root cause analysis to determine where and why the subtraction method produces anomalous results would likely require the code owner(s) to inspect the source code modules. Several factors, including funding, would make such an analysis impractical. DOE neither owns, nor is responsible for, the development of any variation of SASSI. With the existence of multiple proprietary versions of the SASSI code, there is no single entity having sole ownership of the code. To address the near-term use of SASSI, additional verification and validation problems will be developed to ensure the limitations and range of valid assumptions are defined for the use of the modified subtraction method. These additional verification and validation (V&V) problems are described in Issue 4. Sensitivity analyses comparing results from the direct and
modified subtraction methods will be performed to further validate the modified subtraction method. Funding sources to perform a root cause analysis will continue to be explored.

2) **Address the need for a complex-wide assessment of software quality assurance as it relates to SASSI**

EM and NNSA will review the SSQA practices complex-wide as they relate to SASSI for their respective projects identified in the April 8, 2011, Board letter to DOE. DOE developed questions for an information request for these projects. Appendix A contains the draft questions for the information request. EM and NNSA will review the responses to the information request from their respective sites. As needed, EM and NNSA staff will interface with the respondents to complete missing and unclear information. The responses will be evaluated for consistent understanding of the topics and to confirm that the responses are comparable from site to site. Responses will also be reviewed for completeness and evaluated for adequate implementation of DOE’s SSQA requirements. Based upon the responses, EM and NNSA will determine if any onsite visits or reviews are required to further identify impacts of any potential deficiencies on the projects. A report summarizing the information request responses and their evaluation will be prepared and provided to the Board. The summary report will contain any appropriate actions to be taken by EM or NNSA to address the conclusions in the summary report. These activities are included in Action 4.

In late 2010, the Board staff sent requests for information on the use of SASSI to four DOE projects and sites that have recently used, or plan to use, SASSI: the Waste Treatment Plant, Pit Disassembly and Conversion Facility (PDCF), Uranium Processing Facility (UPF), and Los Alamos National Laboratory (LANL). The responses to these requests raised concerns within the Board staff regarding the implementation of safety software quality assurance (SSQA) requirements in the use of SASSI across these projects, which, in part, engendered the April 8, 2011, letter. The Board staff shared the responses from the projects with DOE Headquarters staff (CNS, EM, and NNSA). DOE Headquarters reviewers noted inconsistencies in the responses between one project’s response to a Board staff question and that of another project. Most responses to questions were not detailed enough to determine compliance; however, neither did the responses readily indicate noncompliance.

3) **Address the need for DOE to include outside experts from such organizations as its national laboratories, the nuclear industry, appropriate universities, or the National Academy of Engineering**

The DOE agrees that such outside experts can make useful contributions and has consulted with several of the foremost experts in designing FEMs for SASSI analyses, executing the code, and processing results. These individuals are very experienced in applying SASSI to DOE facilities. Their knowledge and experience, in combination with input from their peers, have been sufficient to define the problems with the subtraction method and devise methods for avoiding them. These experts network regularly with some of the original SASSI developers and other prominent SASSI practitioners, some of whom work for national laboratories and the commercial nuclear industry. On May 16, 2011, DOE staff held a teleconference with Dr.
Farhang Ostadan of Bechtel Corporation, one of the original SASSI developers, to discuss findings and receive feedback. Dr. Ostadan has reviewed the technical report and provided feedback to DOE in early June. The technical report was revised in light of Dr. Ostadan's comments. The dialogue with Dr. Ostadan will continue. DOE has contacted experts in academia regarding their interest and ability to collaborate on the SASSI issues. Prospective collaborations with academic institutions are being explored and will be pursued pending funding availability in fiscal year 2012 and beyond.

DOE has made the technical report available to any interested parties, and will continue to discuss this and any other emerging SASSI issues with other SASSI experts as the additional tasks (enhanced guidance and additional validation and verification problems) are completed. The DOE Natural Phenomena Hazards Workshop, scheduled for October 2011, is a venue at which SASSI experts from industry and academia will be sharing their insights and discuss applications of the code. With adequate participation by other SASSI experts, these periodic meetings can serve as a DOE-supported SASSI users’ forum.

4) Address how guidance related to SASSI can be formally communicated to DOE projects currently in the design stage

The Department is using the attached technical report as guidance and has provided it to all DOE sites with facilities in the design stage that have used or are using SASSI, as well as sites with facilities that used SASSI in the past. The technical report provides 11 sample problems that illustrate conditions under which the subtraction method can yield incorrect results. It provides guidance for performing future analyses and reviewing past analyses that used the subtraction method. Although the report provides some advice for SASSI users performing future analyses, DOE finds that additional guidance and V&V problems will be helpful to SASSI users. The V&V problems currently available are of limited use given the geotechnical complexity of most DOE sites and dimensions of DOE facilities. These new problems will provide future users with greater assurance that SASSI results are reliable for a given site. DOE believes that the larger suite of V&V problems mentioned above will also facilitate SASSI users’ long-term implementation of SSQA requirements.

DOE also plans to create a supplemental guidance document for DOE users that will accompany the additional V&V problems. The guidance will highlight the software functions that need to be verified before executing the code for safety-related design activities. The guidance will also communicate any nuances to executing the code and any other information that DOE finds important for users to consider. Guidance for defining the FEM for SASSI analyses will also be included. The guidance will be peer reviewed by additional SASSI experts, including one or more of the original code developers, if they are willing, as well as knowledgeable SQA practitioners. Regardless of guidance and V&V problem completion, the NNSA facilities with recent or future SASSI analyses, including the Chemistry and Metallurgy Research Replacement (CMRR) and PF-4 facilities at LANL, UPF, and PDCF, have used or will use either the direct or modified subtraction methods in their analyses.

HSS will issue an Operating Experience (OE) report that will describe the issues with the subtraction method and provide an overview of the modified subtraction method and its ability to
avoid the shortcomings of the subtraction method. Furthermore, HSS will make the Safety Software Communication Forum (SSCF) available for posting any future SASSI issues. This system will report on issues, their evolution, and ultimate disposition after review and action by the SASSI experts. Any contractors applying SASSI at their sites will be asked to register with, and make use of, the SSCF.

5) **Provide a detailed schedule for corrective actions**

DOE will take the following actions:

<table>
<thead>
<tr>
<th>Action</th>
<th>Due Date</th>
</tr>
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<tbody>
<tr>
<td>1) EM and NNSA issue requests for information from affected DOE sites/projects on SSQA practices related to SASSI</td>
<td>July 31, 2011</td>
</tr>
<tr>
<td>2) HSS issues OE report describing the subtraction method problems and efficacy of the modified subtraction method</td>
<td>August 15, 2011</td>
</tr>
<tr>
<td>3) Requests for information on SSQA practices related to SASSI due to EM and NNSA Headquarters</td>
<td>September 30, 2011</td>
</tr>
<tr>
<td>4) EM and NNSA complete the review and evaluation of responses from the SSQA information request and generate a summary report. HSS to issue summary report of the SSQA responses to DNFSB</td>
<td>December 30, 2011</td>
</tr>
<tr>
<td>5) DOE includes SASSI in the SSCF</td>
<td>February 29, 2012*</td>
</tr>
<tr>
<td>6) DOE completes additional V&amp;V problems to assist SASSI users</td>
<td>July 31, 2012**</td>
</tr>
<tr>
<td>7) DOE completes supplemental guidance document for SASSI users</td>
<td>September 30, 2012**</td>
</tr>
</tbody>
</table>

*This date is contingent upon the DOE Chief Information Officer’s approval for the release of the SSCF.
**These dates are contingent upon funding levels.

**References**


Appendix A
Draft Questions for SSQA Information Request

Questions for DOE Prime Contractors and Subcontractors:

Software Identification
1. Provide a copy of the safety software inventory list identifying: a) complete name of the SASSI software; b) version identifier; c) provider organization or company name; and d) date of acquisition used for this project. Please ensure the date of the safety software inventory is included. If SASSI is not considered safety software in your project, describe why it does not meet the definition of safety software as described in DOE O 414.1C.

Procurements
2. If SASSI was acquired, identify the organization and describe the process used to obtain your version of SASSI. Provide the procurement documents associated with the acquisition of SASSI.
3. If SASSI is being used by an engineering service provider who owns this software for your site/facility, identify the quality assurance requirements flowed down to the service provider. Provide the procurement, statement of work, and any other contractual agreements.

SQA Work Activities and Procedures
4. Identify all consensus standards; include editions (e.g., ASME NQA-1-2000, IEEE-730-2002, ISO 9000-3-2004) that are related to SASSI on your project.
5. Identify the type of software (e.g., custom or acquired) that SASSI is considered to be in your project.
6. Describe the process for the development, acquisition, and use of SASSI. Additionally, provide a list, including document identifier and title, for all company procedures that apply SASSI in your project.
7. Describe how the 10 safety software work activities in DOE O 414.1C were implemented for SASSI.
8. Describe the contents and provide a list, including document identifier and title, of all documentation associated with SASSI in your project.

Change Management
9. Describe the strategy for managing and controlling the version of SASSI used in your project.
10. If SASSI is characterized as custom software, describe how changes are initiated, evaluated, and approved. Include how changes are controlled prior to approval of the change.
11. Describe the process and documentation maintained for reporting and tracking to resolution any suspected errors related to the use of SASSI. If a problem has been identified, provide the documentation associated with reporting and tracking it.
12. Provide a list of all problems encountered with SASSI along with the investigative and corrective actions taken to resolve those problems, including who or what entity has been consulted to date.

**Verification and Validation**

13. Describe how the test process provides for evaluating technical adequacy through comparison of test results from alternative methods such as hand calculations, calculations using comparable proven programs, or empirical data and information from technical literature.

14. Describe the process for retesting SASSI. Include the criteria used to determine the required periodicity and level of retesting. Describe the circumstances when such testing is necessary.

15. Describe the testing process used to approve SASSI for use.

**Questions for DOE/NNSA Site Offices**

16. Provide a copy of the approved Quality Assurance Program (QAP), with approval signatures, that governs development, acquisition and use of SASSI for this project. If the QAP has not been formally approved, provide documentation of the QAP submittal to the appropriate approval authority, including the QAP submitted. If the QAP is proprietary, provide a copy of DOE approval authorizing its use on your project.

17. Describe the reviews, surveillances, assessments or other oversight activities performed by: a) DOE/NNSA Headquarters, b) field offices, and c) the prime contractor organization, which activities were performed to ensure that the QA activities associated with the development, acquisition, and use of SASSI were implemented in accordance with the QAP or other requirement. Include dates and summary reports for these reviews, surveillances, assessments, or other oversight activities.
U.S. Department of Energy

Soil-Structure Interaction Report

July 2011

Dr. Brent Gutierrez, PE
U.S. Department of Energy
Savannah River Operations Office
Introduction

The System for Analysis of Soil Structure Interaction (SASSI) Code (Refs. 1, 3) has become the de facto industry application used in the analysis of most seismic soil-structure interaction (SSI) problems. For that reason, having and maintaining confidence in the accuracy and applicability of its solution algorithms is essential. As is well known, there are many versions of the SASSI Code, which differ primarily in modeling size capability and execution speed, but which are based on the same flexible volume concept in the original formulation developed by John Lysmer and his doctoral students (Ref. 3). It is important, however, that the user community be aware of the required run parameters (e.g., finite element meshing requirements, solution parameters) and the need to validate computed results for each problem investigated to ensure that the results are valid and appropriate for use in design of the critical facility. In several recent applications, anomalies in computed responses have been noticed, and this has led to investigations to determine the causes.

As described in the correspondence from W. S. Tseng (Ref. 4), the SASSI program uses a method of substructure deletion known as the flexible volume method, commonly known as the "direct method," in which every node within and on the volume of the excavated soil volume is treated as an "interaction node" coupling the free-field soil system and the excavated soil volume. In the late 1990s, a simplified method of substructuring, termed the "subtraction method," was developed (Ref. 5), in which only the nodes lying on the outer perimeter boundary of the excavated soil volume are treated as interaction nodes. Since only the boundary nodes of the excavated soil volume are interaction nodes, the number of interaction nodes for the subtraction method is substantially reduced compared to the direct method, thereby significantly reducing computer resources. The reduction becomes very significant as the soil excavation volume becomes larger, requiring more finite elements in the SASSI model. The SASSI2000 Users' Manual (Ref. 1) states that the subtraction method is the preferred method of analysis. Virtually all SASSI calculations used in the DOE complex in the last 10 years have used the subtraction method for embedded structures.

Since the direct method requires every node in the excavated soil volume to be an interaction node, under free-field ground motion excitations, the excavated soil volume moves in a compatible fashion with the free-field soil system and with the local deformation from the structural loading at every interaction node within and on the boundary of the excavated soil volume. As a result, the direct method could achieve a reasonable simulation for engineering
purposes to the coupled soil-structure system even though finite element formulations are used as approximations to the actual flexibility of the excavated zone. As with other finite element methods to analyze wave propagation problems, it is expected that the finer the finite element mesh and layering, the better the approximation of the wave problem can be obtained. The computed behavior in any particular problem, then, degrades with decreases in mesh refinement in a relatively uniform manner, but not in an unstable manner as is noticed in recent subtraction method SASSI solutions.

Since the subtraction method technique requires only the nodes on the outer perimeter boundary of the excavated soil volume to be interaction nodes, the compatibility of dynamic motions between the free-field soil and excavated soil volume is enforced only at the perimeter boundary where the structural basement nodes and interaction nodes are in common. The actual cause of any possible errors in the subtraction method is not clearly identified as yet; currently, therefore, the extent and severity of the possible local dynamic-response-motion incompatibility for each SSI problem of interest can only be rigorously assessed through systematic comparative studies of individual problems. A modification of the subtraction method herein known as the modified subtraction method (MSM) introduces additional interaction nodes to those used in the subtraction method. Results using the modified subtraction method have been shown to converge with the direct method solution over a larger frequency range than those of the subtraction method. The modified subtraction method adds interaction nodes to the excavated soil volume, often at the ground surface elevation, to the interaction nodes in the subtraction method. The differences between the direct method and the modified subtraction method, however, still remain at those nodes not defined as interaction nodes. Therefore, it can be expected that even the modified subtraction method approach may exhibit instabilities in computed response, as does the subtraction method. As noted in this report, this has been found to occur, but at higher response frequencies. Thus, adequacy of the response must be determined over an appropriate frequency range of interest for any particular problem.

The purpose of this report is to develop a set of general guidelines to assist structural designers in assessing potential numerical instabilities that may occur with the use of the subtraction method.
Discussion and Results
This report examines the applicability of the SASSI subtraction method of analysis relative to the direct method, particularly for embedded structures, to determine if there is a frequency range over which the subtraction method provides satisfactory results for computed in-structure response spectra. Specifically, this investigation considered whether there is an upper bound frequency that can be identified for which the subtraction method should not be used to generate responses for given embedment geometry and soil properties. As a lemma to this, the use of the modified subtraction method appears to raise the threshold of the frequency to a higher absolute value for a given embedment geometry. This issue is believed to be confined to the higher frequency response and primarily impacts in-structure response spectra more so than forces, although this also needs to be verified on a problem-by-problem basis.

Several studies consisting of embedded box-shaped foundations and additional configurations developed by engineers in the Structural Mechanics Section at the Savannah River Site were evaluated. These models were binned according to their complexity in modeling the soil-structure interaction problem as simple, moderate, and complex (see Table 1).

Table 1 SSI Model Complexity

<table>
<thead>
<tr>
<th>Case</th>
<th>Simple Model Bin</th>
<th>Moderate Model Bin</th>
<th>Complex Model Bin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
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<td></td>
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</tr>
<tr>
<td>Case 3</td>
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<tr>
<td>Case 4</td>
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<td>Case 5</td>
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<tr>
<td>Case 6</td>
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<td></td>
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<tr>
<td>Model 1</td>
<td></td>
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<td>Model 3</td>
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<td>PF-4</td>
<td></td>
<td>✓</td>
<td></td>
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<tr>
<td>Stiff Soil Model</td>
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<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

Simple models are those with a simple excavation model and uniform soil layer properties. Moderate models are those with slightly larger excavation model with and without a superstructure and either uniform or varied soil layers. Complex models are those modeling actual facility configurations and soil layers.
Table 2. Modeling Parameters

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Excavated Soil Model</th>
<th>Soil V_s (fps)</th>
<th>Finite Element Size</th>
<th>Site Model</th>
<th>Analysis Performed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50' x 50' x 24' solid concrete box</td>
<td>800</td>
<td>5' x 5' x 6'</td>
<td>44 soil layers @ 800 fps 4 layers at 6' and 40 layers at 2' = 104' deep on elastic half-space</td>
<td>✓</td>
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<tr>
<td>2</td>
<td>50' x 50' x 24' excavated soil volume (800 fps)</td>
<td>30,000</td>
<td>5' x 5' x 6'</td>
<td>44 soil layers @ 30,000 fps 104' deep on elastic half-space</td>
<td>✓</td>
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<tr>
<td>3</td>
<td>100' x 100' x 24'</td>
<td>1,350</td>
<td>10' x 10' x 6'</td>
<td>45 SRS soil layers</td>
<td>✓</td>
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<tr>
<td>4</td>
<td>50' x 50' x 24' SRS props for house model</td>
<td>SRS rigid soil</td>
<td>✓</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>220' x 240' x 50' SRS props</td>
<td>Structural stick model Soil elements</td>
<td>✓</td>
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<tr>
<td>6</td>
<td>220' x 240' x 50' SRS props</td>
<td>Structural stick model and plate elements Soil elements</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cases 1 - 6

These study configurations consisted of rectangular embedded foundations 50 x 50 x 24 feet deep (Figure 1), 100 x 100 x 24 feet deep (Figure 2) and were evaluated for soils with shear wave velocities of 800 feet per second (fps) and 1,350 fps. An irregular foundation of approximately 200 x 200 with 50 feet of embedment was evaluated to typical SRS iterated soil properties, generally in the range of 1,200 fps to 2,000 fps, depending on the particular layer. Table 2 contains more specific modeling parameters.

Simple Models

Case 1
This study looked at the comparison of transfer functions for a site with uniform layer properties. The depth of the site was 104 feet and rested on a uniform half-space with the same properties.
as the layers. Analyses for the subtraction method, modified subtraction method, and direct method were run. The soil shear wave velocity was 800 fps and the embedded structure was a concrete block 50 x 50 x 24 feet deep. The plan horizontal element dimension was 5 feet, and the height of each excavated soil element was 6 feet. Figures 3 to 5 show results for Case 1. Figure 3 compares results between the direct method and modified subtraction method and show that they are in good agreement, even beyond the frequency where a modified subtraction method fundamental frequency occurs at about 21.4 Hz. Figure 4 compares the direct and subtraction methods and shows significant differences from about 15 Hz and above and also shows a major deviation at 16.6 Hz, which is close to the fundamental frequency of the excavated soil block (the 50 x 50 x 24-foot deep soil block with sides and base fixed) at 16.9 Hz determined from an Eigen solution using GT STRUDL.

For both Case 1 and Case 2, the largest element or layer dimension is 6 feet, and for a shear wave velocity of 800 fps, reliable results for frequencies below 26.7 Hz are expected. Figure 5 compares the response spectra from the direct and subtraction methods. In all cases, as shown in Figure 5, the response spectra for this model with a relatively soft soil show acceptable agreement.

**Case 2**
In this study, rigid soil and an embedded 50 x 50 x 24-foot box of material with a shear wave velocity of 800 fps was run to confirm the lowest frequency of the system. The plan element dimension was 5 feet and the height element dimension was 6 feet. The transfer function was compared to a GT STRUDL-computed fundamental frequency to verify the lowest lateral frequency. The result is shown in Figure 6. The unconstrained frequency for the 24-foot embedment, based on the depth of the block and the shear wave velocity for 800 fps soil, is 8.33 Hz; however, the frequency calculated for boundary conditions for the subtraction method using GT STRUDL is 16.9 Hz and compares favorably with the 17.1 Hz peak of the transfer function calculated from the SASSI analysis. It is noted that divergence for the subtraction method does not occur for this case until the frequency associated with the constrained box model; i.e., constraint on the four sides and the bottom is passed.

**Model 2 – Equal Sides and Depth (Cube Excavation)**
The analysis considered a foundation 40 x 40 x 40 feet deep (Figure 7). The walls and base slab are composed of the same materials and have the same properties as Model 1 (see Table 3), with no internal stick, masses, or rigid beams. The mesh size was 4 ft square. The soil profile consisted of uniform soil with a unit weight of 0.120 kips/ft$^3$ (kcf), a S-wave velocity of 875 fps, a
P-wave velocity of 1,462 fps, and a damping ratio of 0.02. The model considers 40 layers four feet thick over a half-space with the same properties as the soil profile. The profile is shown in Table 4.

Table 3. Properties of foundation walls and base slabs

<table>
<thead>
<tr>
<th>Material 1: Slab and Walls:</th>
<th>Material 2: Vertical Structure Beam:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus, $E = 519,120$ ksf</td>
<td>Modulus, $E = 519,120$ ksf</td>
</tr>
<tr>
<td>Poisson's ratio, $\nu = 0.17$</td>
<td>Poisson's ratio, $\nu = 0.17$</td>
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<tr>
<td>Unit weight, $\gamma = 0.15$ kcf</td>
<td>Unit weight, $\gamma = 0.15$ kcf</td>
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<table>
<thead>
<tr>
<th>Material 3: Rigid Beams:</th>
<th>Beam Property 1: Vertical Structural Beam</th>
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<tr>
<td>Modulus, $E = 10,000,000$ ksf</td>
<td>Material 2</td>
</tr>
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</tr>
<tr>
<td>Unit weight, $\gamma = 0.15$ kcf</td>
<td>Shear area, $Av = 340.46$ ft$^2$</td>
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<table>
<thead>
<tr>
<th>Beam Property 2: Rigid Beam</th>
<th>Base Slab Plate Elements</th>
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</thead>
<tbody>
<tr>
<td>Material 3</td>
<td>Material 1</td>
</tr>
<tr>
<td>Area, $A = 400$ ft$^2$</td>
<td>Thickness, $t = 5$ feet</td>
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<tr>
<td>Shear area, $Av = 340.46$ ft$^2$</td>
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<tr>
<td>Moment of Inertia, $I = 13333.3$ ft$^4$</td>
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<table>
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<tr>
<th>Wall Plate Elements:</th>
<th>Elevated Mass = 10,000 kip</th>
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<td>Material 1</td>
<td></td>
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<tr>
<td>Thickness, $t = 5$ feet</td>
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Table 4. Soil Profile 1 Uniform Site

<table>
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<tr>
<th>Layer Number</th>
<th>Thickness (ft)</th>
<th>Unit Weight (kcf)</th>
<th>S-Wave Velocity (fps)</th>
<th>P-Wave Velocity (fps)</th>
<th>S-Wave Damping Ratio</th>
<th>P-Wave Damping Ratio</th>
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<td>0.120</td>
<td>875.0</td>
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</tr>
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<td>875.0</td>
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</table>

The results for Model 2 are shown in Figures 8 to 11. These figures are plotted similarly to the previous model with transfer functions and 5% damped spectra at the center of the base slab. Only X and Z responses are shown. The fundamental frequencies of the excavated soil model, with sides and bottom fixed, are 20.9 Hz in the horizontal direction and 16.4 Hz in the vertical direction. The transfer function confirms the observation in the other models that divergence between the subtraction method and the direct method begins to occur near these frequencies.
Moderate Models

Case 3
Figures 12 to 14 show results from the study with an iterated site profile from the Savannah River Site for an embedded box of 100 x 100 x 24 feet. The soil shear wave velocity averages approximately 1,350 fps over the 24-foot depth. Figure 12 compares results between the direct method and MSM and shows that they are in good agreement.

Figure 13 compares the transfer functions obtained from the use of the direct method and subtraction method and shows significant differences from about 18 Hz and above. Figure 14 compares the resulting response spectra. An Eigen solution performed with GT STRUDL using a shear wave velocity of 1,350 fps resulted in a fundamental frequency of 18.8 Hz (constrained frequency as seen in Figure 15).

The SRS soil layers for the first 24 feet varied in each layer; 1,345 for layers 1 and 2, 1,328 for layer 3, and 1,286 for layer 4. This would explain the difference between the SASSI frequency at 18 Hz and the GT STRUDL-determined frequency at 18.8 Hz. The unconstrained frequency for a 24-foot depth at a shear wave velocity of 1,350 fps is 14.1 Hz. As seen, the divergence between the direct method and subtraction method in each case begins to appear between the free-field frequency and the constrained frequency. The largest element or layer dimension is 10 feet, and, for a shear wave velocity of 1,350 fps, reliable results for frequencies below 27 Hz are expected.

Case 4
In this study, an analysis was performed of an embedded solid concrete structure 50 x 50 x 24 feet deep – element size 5 feet by 5 feet, 6-foot layers. The backfill (HOUSE model) is rigid and surrounded by the SRS site soil profile with shear wave velocity of 1,350 fps for the top layers. The fundamental frequency of the excavated soil constrained along its sides and bottom is 28.8 Hz.

Figure 16 shows that the transfer functions at Node 1150, which is located in the middle of the top surface, are almost identical for the direct method, subtraction method, and MSM up to 25.0 Hz. The transfer function differs for the subtraction method above 25.0 Hz, which is outside the cutoff frequency for the model, and shows a spike at approximately 33.0 Hz. Figure 17 shows that, for practical purposes, the in-structure response spectra at Node 1150 are the same for direct, subtraction, and MSMs for the soil at the Savannah River Site.
Model 1 – Rectangular Excavation with Internal Stick
The analysis considered a foundation that is 50 x 100 x 20 feet deep for two mesh sizes. A stick representing the superstructure with distributed mass along the height and lumped mass at top is included to have a structural frequency of 9.6 Hz in the X-direction, 10.1 Hz in the Y-direction. Both coarse and fine mesh models use the same geometry as shown in Figures 16 through 19. The foundation is 20 feet deep divided into five layers, 50 feet wide in the X-direction, divided into six elements in the coarse mesh and fourteen elements in the fine mesh, and 100 feet wide in the Y-direction, divided into twelve elements in the coarse mesh and twenty-six elements in the fine mesh. The structure consists of the base slab, sidewalls, vertical structural beam, elevated mass, and rigid beams around the perimeter at the top of walls and connecting to the vertical beam. Grade is considered to be at the top of the modeled perimeter walls. The vertical beam extends from the center of the slab to the rigid beams at grade and up to the elevated mass 20 feet above grade. The materials, masses, and properties used in this model are contained in Table 3 and free-field soil properties are presented in Table 4.

Excavated soil elements use the same mesh sizing as the outer walls and fill the entire embedded region, connecting to the structure at the perimeter nodes (identical to nodes on plate elements). For the subtraction method, the interaction nodes are selected around the sides and bottom of the excavation, consistent with the SASSI2000 User’s Manual (Ref. 1). For the direct method, all nodes on the excavation elements are interaction nodes. For the MSM, interaction nodes consist of all nodes on the outside faces of the excavated soil, including the top surface.

The soil profile consists of uniform soil with a unit weight of 0.120 kcf, an S-wave velocity of 875 fps, a P-wave velocity of 1,462 fps, and a damping ratio of 0.02. The model considers 40 layers four feet thick over a half-space with the same properties as the soil profile. The profile is shown in Table 4.

For each soil case and direction of motion, the transfer functions (Figures 22, 24, 26, 28, 31, 33, and 37) and response spectra (Figures 23, 25, 27, 29, 30, 32, 33, 35, and 36) at the center of the base slab are plotted comparing the response from each of the three analysis methods. The plots also include a vertical dotted line representing the frequency of the excavated soil column \( f_{sc} = \frac{V_s}{4H} \) (where \( V_s \) is the shear wave velocity of the soil column and \( H \) is the excavation depth) and a vertical solid line representing the maximum passing frequency of the excavated soil \( f_p = \frac{V_s}{5L} \) (where \( V_s \) = lowest shear wave velocity of any soil layer and \( L \) is the largest
horizontal or vertical dimension of an excavated soil element). Transfer functions are shown on a linear scale with black dots showing the calculated transfer function values and lines showing interpolated values. The transfer function curves also include a vertical line representing the first significant mode of the excavated soil in the direction of interest. The results of Eigen value calculations were also performed considering nodal fixity at interaction nodes, removing the structure, and setting excavated soil properties equal to free field and are listed on the transfer function plots.

Transfer functions and response spectra results for the uniform soil coarse mesh are shown in Figures 22 to 27. Direct and subtraction methods differ above 15 Hz, and the MSM produces results very similar to the direct method. Response spectra are not largely affected due to the low energy at the frequencies with transfer function differences.

Transfer functions and response spectra results for the uniform soil fine mesh are shown in Figures 28 to 36. A comparison of transfer function results for the coarse and fine mesh is shown in Figure 37. The results for the finer mesh are similar in behavior, showing deviations at approximately the same frequencies, but differ in magnitude to the coarse mesh results. Response spectra considering the higher frequency input of the eastern site input motion show that the spectra can be affected by the differences in transfer function in the higher frequencies, which may influence equipment qualification.

A time history with a spectral peak near 0.45g at about 1.5 Hz representing a western U.S. site is used as input motion. To demonstrate that high-frequency transfer functions can make a difference, a time history with a spectral peak around 0.9g at about 25 Hz representing an eastern U.S. site was also considered for the fine mesh model. The input motion spectra are shown along with response spectra in Figures 29 and 30.

**Model 3 – Rectangular Excavation Revisited**

Model 1 was revised to incorporate attributes to demonstrate that the subtraction method response differences can affect structural responses of interest. Model 3 is shown in Figure 38. The mesh was revised to a 6.25-foot element dimension horizontally and vertically, and the soil layers revised to $V_s = 600$ fps, $V_p = 1191$ fps. Fifteen 10 kip masses are added to nodes connected to the center of the base mat by springs with stiffnesses selected to cover a range of frequencies from 5 to 18 Hz in 1 Hz intervals. The results at the center of the base slab and at three of the masses on springs with the subsystems having natural frequencies of 12 Hz, 13 Hz, and 14 Hz were examined (Figure 38).
The results showed a similar response to Model 1, with the onset of divergence between the subtraction and direct methods at lower frequency. Figures 39 and 40 present corresponding 5% damped response spectra computed at the top of the excavated volume. The masses that respond near the frequencies of divergence show responses substantially different for the two methods (Figures 41 through 44). Once again, the fundamental frequencies of the excavated soil are shown on the figures showing the transfer functions. The modified subtraction method responds similarly to the direct method.

**Plutonium Facility - 4 (PF-4) Model**

As shown in LA-UR-10-05302 (Ref. 2), differences in the SSI computations performed with the subtraction method, which were measured and compared with results from the direct method of analysis, have been found to occur at frequencies close to the first mode frequency of the excavated soil zone. Some results shown in Ref. 6 indicate significant differences at frequencies below the mesh cutoff frequency. Figure 45 presents a sample finite element mesh to model the excavated zone. It is a reasonably uniform mesh over the plan area of the structure of the PF4 facility at LANL. Figure 46 is a comparison plot of transfer functions computed from the subtraction method and MSM, and indicates that the subtraction method results become unstable at frequencies well below the mesh cutoff frequency. Figures 47 through 50 show the transfer functions and response spectra obtained from a particular node in the model. Currently, there is no clear relation between the magnitude of the differences that may be encountered and the characteristics of the finite element mesh and SSI problem.

**Complex Models**

**Case 5 and Case 6**

The Savannah River Site Structural Mechanics Engineers completed studies to compare the different solution methods for SASSI with some of the existing models used for past analyses.

**Case 5**

In this case, a stick model representing a building structure is attached to a stiff wall structure that represents the embedded part of the building. The embedded part is modeled with plate elements that are very stiff to reflect restraint of interior walls and floors that are not included as elements in the model. Figure 51 shows the model, and Figure 52 shows the results, which compare the MSM and the direct method for the transfer functions and some of the response spectra for the stick model attached to a rigid plate embedded structure. The cutoff frequency for this study was 15 Hz (i.e., the highest frequency that can reliably be transferred by the soil layers). The calculated fundamental frequency of the excavated 220 x 240 x 50-foot deep soil
geometry is approximately 9 Hz for the embedment with the top surface free, and 13-14 Hz with a fixed top surface. The differences in the transfer functions for the direct method and MSM begin to be observed at approximately 15 Hz. There are no significant differences shown for the response spectra, which is to be expected since the cutoff frequency is 15 Hz.

Case 6
In this case, the comparison is for a more detailed embedded building model made of plate elements. Figure 53 shows the model, and Figure 54 shows the results. Note the first lateral frequency of the excavated soil geometry: a block approximately 220 x 200 x 50 feet deep, constrained on the sides and bottom, is approximately 9 Hz for subtraction and 13-14 Hz for the MSM. The calculated fundamental frequency of the excavated soil geometry is approximately the same as in Case 4; 9 Hz for the embedment with the top surface free and 13-14 Hz with the top surface fixed. The results show insignificant differences between the MSM and the direct method for the transfer functions and response spectra for the plate model. However, similar to the other cases, there is divergence in frequency ranges higher than the fundamental frequency of the excavated soil block. The deviations may be associated with the element width, which, in this case, is 12-13 feet in the horizontal direction, thereby indicating unreliable results above 13 Hz. The cutoff frequency for this study was 15 Hz.

Stiff Soil Model
For this model, Figure 55, a finite element mesh of the excavated zone was developed that has a transmission or cutoff frequency, defined as \( \frac{Vs}{5H} \), set to about 50 Hz. Transfer function calculations were made using the direct method, the MSM, and the subtraction method, with the MSM adding surface nodes to the set of interaction nodes used for the subtraction method. Figure 56 compares transfer functions between the subtraction method and the direct method. As may be noted, the subtraction method becomes unstable at about 27 Hz, with the direct method performing uniformly up to the 50 Hz mesh transmission frequency. It is clear that the subtraction method instability at 27 Hz is independent of the cutoff frequency. Figure 48 presents a similar comparison using the modified subtraction method in place of the subtraction method. This does not show any instability in the computed response.

Observations from Available Studies
It has been postulated that the natural frequency of the excavated volume may play a role in the instability of the subtraction method. To address this effect, the natural frequency of the substructure volume was determined for a number of different sets of interaction node
configurations (Ref. 8). The first and least refined is the subtraction method set, where each node along the soil model side periphery and base is defined as an interaction node. A second set has every other surface node defined as an additional interaction node. In a third set, every other node of the excavated volume at a depth of 11.5 feet is defined as an interaction node. The frequency of each excavated volume with interaction nodes as defined above is determined by performing a sine sweep analysis, imposing unit displacements on each interaction node. As the number of interaction nodes tends towards the total number of interaction nodes in the direct method, the frequency of the excavated volume tends toward infinity, as all nodes in the direct method are restrained and unable to vibrate. It is desirable, of course, to restrain the excavated volume in such a way that its lowest frequency is higher than the highest frequency of interest in the response analysis.

Figure 55 presents a plan view of the mesh of the excavated volume used in the study of Reference 8. Figure 56 presents a comparison of transfer functions for the three excavated volumes mentioned above. The frequency from the subtraction method occurs at the instability frequency noted in Figure 47. For the MSM with every other node of the surface defined as an interaction node, the volume frequency increases to over 50 Hz, but continues to occur at a frequency of about 55 Hz. The solution for the MSM with every other node at Elevation -11 indicates a lowest frequency of about 75 Hz. By reviewing the computed transfer functions from the various calculations, it can be seen that both of these MSM solutions will then satisfy the project criterion of acceptable responses to 50 Hz.

Solutions for embedded structures can be generated from SASSI using the subtraction method, the MSM, or the direct method. The current SASSI Theoretical Manual (Ref. 7) and SASSI Users' Guide (Ref. 1) describe the subtraction method and the direct method. The direct method is a more computationally intensive analysis methodology based on the finite element method of analysis. The subtraction method is based on the modification to the equation of motion that results in a smaller set of interaction nodes. Both equations of motion are derived from the same principles. However, the numerical matrix formulation needed in the subtraction method based on the work of C. C. Chin (Ref. 5) may cause instability under certain parameters used in the model. The current SASSI Users' Guide (Ref. 1) states "...use of the subtraction method results in a significantly smaller set of interaction nodes without loss of any accuracy and is recommended as a primary method of impedance analysis." The MSM (additional interaction nodes in the excavated soil volume on its upper surface) is not discussed in the SASSI Users' Guide, and has been only recently mentioned in response to the recent discrepancies noted in
References 2, 4, 6, and 8. It should be noted that there is currently no formal reference to the MSM. An independent analysis of the problems evaluated in Reference 6 indicated that the use of the MSM significantly improved the results as compared to those from the subtraction method. As reported in Reference 11, the discrepancies reported in Reference 2 were shown to be effectively eliminated using the MSM (according to the author of Reference 11). It has also been noted from other sensitivity studies that selecting any set of nodes as additional interaction nodes does not necessarily lead to a uniform improvement or convergence to the correct solution defined by the direct method analysis.

Conclusions
This report presents a number of examples to show the sensitivity and frequency limitation of the subtraction method for some applications. The intent of the report is to provide guidelines to assist in locating potential sensitivities and in evaluating impacts on design parameters.

In general, the frequency associated with the excavated soil portion of the model is a limit for the accuracy of transfer functions generated using the SASSI subtraction method. For example, for an SSI analysis performed using the subtraction method with its four vertical sides and its base used as interaction nodes, the fundamental frequency of the excavated soil block would be the maximum frequency for the SASSI solution. Performing the same analysis with the MSM, which adds the upper surface to the set of interaction nodes, a higher fundamental frequency is obtained, but is still considered the maximum frequency for the SASSI solution. Thus, when performing an SSI analysis using either the subtraction method or MSM on an embedded structure, the fundamental frequency of the excavated soil volume needs to calculated and the applicability of the obtained responses limited to those lower than that fundamental frequency.

In addition, some SASSI analysts have performed a series of analyses where the number of interaction nodes is increased in each subsequent analysis. The results are then compared with those from previous iterations to determine when changes occur. These analyses serve as the basis for selecting the final set of interaction nodes used.

Transfer functions and response spectra obtained using the subtraction method show deviations from results obtained using the direct method of analysis at frequencies related to the properties and size of the finite element mesh (effective frequency characteristics) as well as response characteristics of the problem investigated (site frequencies). At the present time, it appears that these differences are primarily limited to:

• the higher frequencies controlling the in-structure response spectra,
• the magnitude of these deviations, and
• the characteristics of the finite element mesh or dynamic properties used to investigate the particular SSI problem.

Therefore, further study is required to understand and resolve these differences. In the interim, as stated in the recommendations, the subtraction method and the MSM should be used with caution if the direct method is not feasible.

As with any finite element elastic analysis of wave propagation, the adequacy of the computation depends on the element discretization used to obtain adequate results over the specified frequency range of interest. The guidance provided in the SASSI2000 manuals (Refs. 3 and 4) recommends that, for a given finite element dimension, the elements should be sized to transmit a frequency of at least $V_s/5H$, where $H$ is the largest element dimension in the excavated volume considering all three dimensions of the element. For frequencies beyond this cutoff frequency, it would be expected that the computation’s accuracy would gradually decrease, as the element cannot transmit these higher frequencies but gradually tends to act as a rigid body at higher frequencies. The loss of accuracy depends on the complexity of the SSI problem and the non-uniformity of the soil profile modeled as part of the SSI problem. This is standard computational behavior that has been noted in finite element wave propagation problems for many years. In the analyses presented in some cases in this report, the gradual decline is not apparent; rather, there are dramatic deviations between the subtraction method and the direct method.

As shown in LA-UR-10-05302 (Ref. 2), differences in the SSI computations performed with the subtraction method and with the direct method have been found to occur at frequencies close to the first mode frequency of the excavated soil zone. Some results indicate significant differences at frequencies below the mesh cutoff frequency (Ref. 6). Currently, there is no clear relationship between the magnitude of the differences that may be encountered and the characteristics of the finite element mesh and SSI problem. However, the guidance relative to the fundamental frequency of the excavated soil volume discussed earlier should be followed.

The measure of the differences in the SSI computations performed with the subtraction method, the MSM, and the direct method has generally been defined in terms of large exceedances between transfer functions. The direct method is the more robust solution for a given soil layering and finite element mesh. Transfer functions are generally agreed to be the most sensitive measure of the differences between the subtraction and direct analysis methods.
Similar differences have been noted in the computation of soil impedances for a rigid, massless foundation at frequencies close to the mesh size limit. The consequences of these differences in particular problems on design parameters such as in-structure response spectra, element forces, and moments may not be as profound as indicated by the transfer functions, but still need to be evaluated on a case-by-case basis to ensure an adequate level of conservatism in the design. In many cases, a dip in the transfer function occurred at a frequency immediately below the frequency of the instability (see Figure 46, for example), followed by the peak. Therefore, the computed response using the subtraction method is not always conservative.

At this time, there is no formal reference to the MSM. An independent MSM analysis of the problems evaluated in Ref. 6 indicated that its use significantly improved the results compared with those using the subtraction method. It has been noted from other sensitivity studies that selecting any set of nodes as additional interaction nodes does not necessarily lead to a uniform improvement or convergence to the correct solution defined by the direct method analysis.

Response spectra are not as sensitive to the instability of the subtraction method as are transfer functions; however, the differences are still evident. Therefore, for soil-structure interaction analyses of embedded DOE structures, the evaluation of the sensitivity of the results should be based on design parameters as guided by the observed exceedances in the transfer functions.

While it has been shown that the use of the MSM may be appropriate, alternative schemes that allow a reduced number of interaction nodes, (i.e., less than using the full boundary of the excavated soil but greater than the original subtraction method) are feasible and acceptable for conducting soil-structure interaction analyses of embedded DOE structures, provided that sufficient justification is included in the SSI calculations.

The results from these studies suggest that the direct and subtraction methods differ in responses at higher frequencies for cases of wide shallow excavations, with the direct method producing more reasonable results. Analyses conducted using the modified subtraction method with interaction nodes along the surface more closely matches the results calculated using the direct method.

In the Model 1 and Model 2 cases described above, the observed exceedances in the transfer functions did not significantly impact the response spectra for the western U.S. time history input motions. This is primarily because the time history input used contained very little high
frequency energy. The transfer function exceedances have a much greater effect for eastern U.S. input motions however, where the spectral peak is in the region of transfer function exceedance. Model 3 results demonstrate that, for some conditions, the subtraction method can produce conservative or unconservative results relative to the direct method, even with eastern US time history motions. Therefore, it is necessary to evaluate and document the effect of the high frequency input when considering using the subtraction method for embedded DOE structures.

Recommendations
SASSI is a specialized program that has limited supporting documentation, limited verification and validation documentation, and limited technical support from organizations commercially distributing this program. Therefore, special attention and experience are required to perform SASSI analyses. While the subtraction method works for certain cases, it may be sensitive for other cases depending on the soil and structural properties and frequencies of interest. The following recommendations are proposed for facilities already analyzed or to be analyzed using SASSI, particularly if the subtraction method or MSM is used.

Recommendations for Facilities Already Analyzed with SASSI
An analyst with sufficient experience (at least five years) with the SASSI software should evaluate the seismic analyses of each embedded or partially embedded facility performed that used SASSI with the subtraction method especially when one or more of the following conditions are present:

- The transfer functions exhibit peaks and valleys not justified by expected structural and SSI responses. In examining the transfer functions, the frequency limitations discussed below must be considered.
- The frequency of interest for seismic responses is above the frequency of the excavated soil layer \( f_{SL} = \frac{V_s}{4H} \), where \( V_s \) is shear wave velocity and \( H \) is depth of excavated volume.
- The frequency of interest for seismic responses is near or above the frequency of the excavated volume \( (f_{EV}) \), modeled by a solid finite element model fixed at interaction nodes.
- The input motion has significant energy above the frequency of excavated soil, \( f_{SL} \).
- The seismic responses of interest include ISRS with significance above \( f_{SL} \).

If the evaluation does not conclusively determine that the design parameters obtained using the subtraction method are acceptable, the analyses should be performed using the direct method.
or the MSM to confirm the previous results. If necessary, smaller sub-models may be used for this purpose. If the MSM is used, the applicability of the number and location of the added interaction nodes needs to be evaluated to demonstrate that all frequencies of interest are included and result in responses appropriate for engineering accuracy.

**Recommendations for performing future seismic analyses for facilities using SASSI**

Analysts with sufficient experience with SASSI should perform the new SSI analyses of facilities using SASSI and fully consider the following recommendations:

- For embedded or partially embedded structures, use the direct method if feasible. Until the cause of the subtraction error is identified, the subtraction method can be used as long as additional evaluations suggested below are made to ensure the design parameters are not impacted.
- An experienced analyst must review the transfer functions for evidence of anomalous response such as that depicted in Figure 4 and other figures to determine if the anomalies are in the structural frequency range of interest.
- Analysts need to consider the frequency range of interest, excavated soil layer frequency $f_{SL}$, and energy content of the input time history in the frequency range of interest.
- Sensitivity analyses should be performed when the subtraction or the MSM is used. Sensitivity analyses could include:
  - rerunning the problem with additional interaction nodes such as the MSM and comparing the transfer functions and the in-structure response spectra
  - use symmetry conditions to investigate behavior of smaller models with similar characteristics or applicable sub-models.
- Further studies are required to resolve the differences between the subtraction method and direct method, and, if feasible, to modify the code to increase the reliability of the subtraction method to be applied to a broader set of problems.

**Recommendation for Verification and Validation**

- The SASSI software should be fully verified in accordance with an approved quality assurance program. A set of generic verification and validation problems should also be developed and provided to DOE for review.
- Some design firms have changed the program and added additional features for their use. For this reason, an appropriate set of verification and validation problems needs to be developed by the design firms to be tested and made available for DOE review.
verification problems must be executed on the same computer platform used for the production analysis.

• The SASSI software also needs to be verified and validated for project-specific features; for example, very soft or stiff soil conditions, or unusual embedment geometry that may deviate from the expected norm for which the software was originally intended.

• Additional validation and verification documentation for each project-specific application of SASSI may be required. Because of its specialized focus, the project needs must be reviewed by an experienced SASSI user in addition to a QA reviewer.

Acknowledgements

The author gratefully acknowledges analyses provided by, and technical discussions with, Dr. Said Bolourchi, PE, Dr. Carl Costantino, PE, and Mr. Frederick Loceff, PE.
Appendix A: References


6. G. Mertz, "Effect of the Subtraction Error in DOE Nuclear Facilities," personal correspondence, 8 November 2010


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Blue curves are modified with additional nodes at -11m defined as excavated